International Journal of Engineering and Applied Sciences (IJEAS)
ISSN: 2394-3661, Volume-3, Issue-9, September 2016

Wireless Video Caching in Heterogeneous Networks: AStackelberg Game Approach

Mutangana Eugene, AbdeldimeM.S.Abdelgader, Qian yu wen

Abstract— **This paper, considers a marketed small-cell caching system comprising of a Network Service Provider (NSP), a number of Video Retailers (VR), and Mobile Users (MU). The NSP rents its SBSs to the VRs intending benefits. Stackelberg game framework is used for addressing the SBSs as a particular type of resources. The MUs and SBSs as autonomous Poisson Point Processes (PPP) are used to develop the probability of the particular event that an MU receives video of its alternative straight from the memory of an SBS through stochastic geometry theory. Furthermore, a Stackelberg game is developed to maximize the average benefit of the NSP and the VRs. We look into the Stackelberg game balance by solving a non-convex optimization job. Therefore, based on game theoretic framework, we split light on the four important factors with respect to their relationship: Optimal pricing of renting an SBS, SBSs allocation among the VRs, Caching size of the SBSs, and the quality dispersion of the VRs. Monte-Carlo simulation show that our stochastic geometry-based analytical results, nearly match the empirical results. Mathematical results are also plied for measuring the intended game-theoretic framework through demonstrating its efficiency on pricing and resource assignation.**

Index Terms— **Heterogeneous cellular networks, Small-cell caching , Stackel berg game, Stochastic geometry**

I. INTRODUCTION

Internet data traffic is anticipated to increment exponentially in the next decade lead by a distributing growth of MU parallel to their bandwidth consumption in mobile applications. It has been proved that on-demand MUs live video caused attention to the advancement of Tele-traffic over mobile networks [1]. In addition, a number of repetitious exploit of pop videos from the MUs, e.g. online blockbusters, and leads to extra video transmissions. The extra data transmissions can be cut down using caching technologies into intermediate network storage nodes [1, 2]. The caching technology contributes video capacity nearer to the MUs which helps extra data transmissions through redirecting the transferring requests to the intermediate storage nodes.

In general, wireless data caching comprises of two phases: Data positioning and Data deliverance [3]. Firstly, Data positioning phase, big videos are cached into storage nodes on off-peak turns, as in Data deliverance phase, videos called for are delivered from local caching system to MUs. Previous researches moved on the caching solutions of the Device-to-Device (D2D) and Wireless Sensor networks (WSN) [4 –6]. In particular [4], a caching scheme was introduced for a D2D based mobile network operating on the MUs' caching of big video capacity. Consequently, in D2D cluster size was optimized for cutting down video downloading delay using caching scheme. In [5, 6], a novel caching schemes for WSN, the protocol model of [7] was introduced. Because small-cell embedded architectures will rule in future heterogeneous networks (HetNet) [8–13], small-cell caching establishes a promising result for Het Nets. In [14], the small-cell caching is investigated in the context of stochastic networks. The average performance is developed via stochastic geometry, where the distribution of network nodes are modeled by Poisson point process (PPP)[15,16].

In this paper, we suggest a marketed small-cell caching scheme comprising an network service provider (NSP), video retailers (VRs) and mobile users (MUs). We optimize such a scheme inside the Stackelberg game framework by considering the SBSs as a particular type of resources for the role of video caching. Furthermore, Stackelberg game is a game that comprises a leader and a number of viewers competing with one another with respect to certain resources as indicated in [14]. The leader comes in at first and the viewers follows. However, in our scheme, we look at the NSP as the leader and the VRs as the viewers. The NSP fixes the cost of renting an SBS, whereas the VRs deal one another for renting SBSs partition. Consequently, this paper implements the first optimization caching system based on game theory. In detail, our tasks are; 1) modeling the MUs and SBSs in the network differently as links to PPP[17] . Based on this network model, we demonstrate an efficient MU video downloading process based on stochastic geometry theory probability over accessed videos directly from the SBS storage; 2) developing a productive caching model with the NSP and VRs benefits from SBSs renting. 3) Suggesting maximized average profit from NSP and the VRs through Stackelberg game framework. With theoretic framework, we look into different pricing strategies whereby the price charged to unlike VRs changes. 4) Through solving a non-convex optimization problem, we checked into the Stackelberg balance of this scheme and the optimal solution is linked to each SBS storage size and the quality dispersion of the VRs. 5) same pricing scheme were considered. Although same pricing scheme is substandard to the different NSP's benefit, we found that same pricing is capable of reducing more backhaul costs.

The rest of this paper is organized as follows. We describe the system model in Section II and establish the related profit model in Section III. We then formulate Stackelberg game for our small-cell caching system in Section IV. In Section V, we investigate Stackelberg equilibrium for the no uniform pricing scheme by solving a non-convex optimization problem, while

Mutangana Eugene, School of Electronic and Optical Engineering, Nanjing University of Science and Technology, Nanjing 210094, China, Tel (+86)15205174316.

Abdeldime M. S. Abdelgader, Department of Electrical and Computer Engineering, Karary University, Khartoum, 12304, Sudan. Tel (+86)13584003982

Qian yu wen, School of Electronic and Optical Engineering, Nanjing University of Science and Technology, Nanjing 210094, China, Tel(+86)18936030253

in Section VI, we further consider the uniform pricing scheme. Our simulations and numerical results are detailed in Section VII, while our conclusions are provided in Section VIII.

II. SYSTEM MODEL

We consider a commercialized small-cell caching system consisting of an NSP, V *VRs*, and a multiple of MUs . Let us denote by L the NSP, by $V= \{v_1, v_2, \dots, v_V\}$ the set of the VRs, and by M one of the MUs. In such a system, the VRs wish to rent the SBSs from L for placing their videos .Both the NSP and each VR aim for maximizing their profits. There are three stages in our system. In the first stage, the VRs purchase the copyrights of popular videos from video producers and publish them on their web-sites. In the second stage, the VRs negotiate with the NSP on the rent of SBSs for caching these popular videos. In the third stage, the MUs connect to the SBSs for downloading the desired videos. We will particularly focus our attention on the second and third stages within this game theoretic framework.

A. Network Model

Let us consider a small-cell based caching network consisting of the MUs and the SBSs owned by L, where each SBS is deployed with a fixed transmit power P and the storage of Q video files. Assume that all the SBSs transmit on the channels orthogonal to those of the macro-cell base stations, and thus there is no interference incurred by the macro-cell base stations. Also assume that these SBSs are spatially distributed according to a homogeneous PPP (HPPP) Φ of intensity λ. Here, the intensity λ represents the number of the SBSs per unit area. Furthermore, we model the distribution of the MUs as an independent HPPP Ψ of intensity ζ. The wireless down-link channels spanning from the SBSs to the MUs are independent and identically distributed (*i.i.d.*), and modeled as the combination of path-loss and Rayleigh fading. Without loss of generality, we carry out our analysis for a typical MU located at the origin. The path-loss between an SBS located at x and the typical MU is denoted by $\|x\|^{-\alpha}$, where α is the path-loss exponent. The channel power of the Rayleigh fading between them is denoted by h_x, where $h_x \sim \exp(1)$. The noise at an MU is Gaussian distributed with a variance σ^2. We consider the steady -state saturated network, where all the SBSs are powered on and keep transmitting data for serving Their MUs.Hence, the received signal-to-interference-plus-noise ratio (SINR) at the typical MU from an SBS located at x, can be expressed as

$$\rho(x) = \frac{Ph_x\|x\|^{-\alpha}}{\sum_{x' \in \Phi \backslash x} Ph_{x'}\|x'\|^{-\alpha} + \sigma^2} \quad \textbf{(1)}$$

Where the numerator $Ph_x\|x\|^{-\alpha}$ represents the received *signal* power at the origin from the SBS located at X.

The denominator $\sum_{x' \in \Phi \backslash x} ph_{x'}\|x'\|^{-\alpha} + \sigma^2$ is the sum of interference caused by SBSs from NSPs.The typical MU is considered to be "covered" by an SBS located at x as long as $\rho(x)$ is no less than a pre-set SINR threshold δ, i.e., $\rho(x) \geq \delta$.

B. Video Popularity and preferences

We now model the popularity distribution, i.e., the distribution of request probabilities, among the popular videos to be cached. Denote by $\boldsymbol{F} = \{f_1, f_2, \dots, f_N\}$ the file set consisting of N video files, where each video clip that is frequently requested by MUs. The popularity distribution among $\boldsymbol{F}$ is represented by a vector

$\mathbf{t} = [t_1, t_2 ; \cdots ; t_N]$. That is, the MUs make independent requests of the n-the video f_n, $n = 1, \dots, N$, with the probability of t_n.

Generally, $\mathbf{t}$ can be modeled by the Zipf distribution [18] as

$$t_n = \frac{1/n^\beta}{\sum_{j=1}^{N} 1/j^\beta} \quad (1)$$

Where the exponent β is the positive value, characterizing the video popularity. A higher α correspond a higher content reuse, where the most popular files account for the majority of the download request. From Eq. (2), the file with a smaller n corresponds to a higher popularity.

C. Small-Cell Caching Process

In this section, we introduce the process of our small cell caching system. In the first stage, each **VR** V purchases the N popular videos in $\boldsymbol{F}$ from the producers, and publishes these videos on its web-site. In the second stage, the VR negotiate with the NSP L for renting these SBSs. As L leases its SBSs to multiple VRs, we denote $\tau = [\tau_1, \tau_2, \dots, \tau_v]$ the fraction vector, where τ_v represents the fractions of the SBSs that are assigned to $v_u \forall_u$. We assume that the SBSs rented by each VR are uniformly distributed. Hence, the SBSs that are allocated to v_u can be modeled as a "thinned" HPPP Φ_v with intensity $\tau_v \lambda$ .The data placements of the second stage commence during network off-peak time after the VRs obtain access to the SBSs. During the placements, each SBS will be allocated with one of the F FGs. Generally, we assume that the VRs do not have the *a priori* information regarding the popularity distribution of $\boldsymbol{F}$. This is because the popularity of videos is changing periodically, and can only be obtained statistically after these videos quit the market. It is clear that each VR may have more or less some statistical information on the popularity distribution of videos based on the MUs' downloading history. However, this information will be biased due to limited sampling. In this case, the VRs will uniformly assign the F FGs to the SBSs with equal probability of $\frac{1}{F}$ for simplicity. We are interested in investigating the uniform assignment of video files for drawing a bottom line of the system performance. As the FG s are randomly assigned,

the SBSs In Φ_v that cache the FG $\mathcal{G}_f$ can be further modeled as a "more thinned" HPPP $\Phi_{v,f}$ with an intensity of $\frac{1}{F} T_v \lambda$. In the third stage, the MUs start to download videos. When an MU M requires a video of $\mathcal{G}_f$ from, it searches the SBSs in $\Phi_{v,f}$ and tries to connect to the nearest SBS that covers M. Provided that such an SBS exists, the MU M will obtain this video directly from this SBS, and we thereby define this event by $\varepsilon_{v,f}$. By contrast, if such an SBS does not exist, M will be redirected to the central servers of v_u for downloading the requested file. Since the servers of v_u are located at the backbone network, this redirection of the demand will

trigger a transmission via the back-haul channels of the NSP L, hence leading to an extra cost.

III. PROFIT MODELING

We now focus on modeling the profit of the NSP and the VRs obtained from the small-cell caching system. The average profit is developed based on stochastically geometrical distributions of the network nodes in terms of per unit area times unit period ($/UAP$), $e.g., /month.km^2$ *A. Average Profit of the NSP* For the NSP L, the revenue gained from the caching system consists of two parts: 1) The income gleaned from leasing SBSs to the VRs 2) The cost reduction due to reduced usage of the SBSs' back-haul channels. First, the leasing income$\overline{UAP}$ of L can be calculated as

$$S^{RT} = \sum_{j=1}^{N} \tau_j \lambda s_j \quad (3)$$

Where s_j is the price per unit period charged to v_j for renting an SBS. Then we formulate the saved cost/UAP due to reduced back-haul channel transmissions. When an MU demands a video in $\mathcal{G}_f$ from v_u .

Theorem 1: The probability of the event $\varepsilon_{vf}, \forall v, f$, can be expressed as

$$Pr(\varepsilon_{vf}) = \frac{\tau_v}{C(\delta,\alpha)(F-\tau_v)+A(\delta,\alpha)\tau_v+\tau_t} \quad (4)$$

Where $A(\delta,\alpha) \triangleq \frac{2\delta}{\alpha-2}\ {}_2F_1\left(1,\ 1-\frac{2}{\alpha};2-\frac{2}{\alpha};-\delta\right)$

And $C(\delta,\alpha) \triangleq \frac{2}{\alpha}\ \delta^{\frac{2}{\alpha}}\ \mathrm{B}\left(\frac{2}{\alpha},1-\frac{2}{\alpha}\right)$. Furthermore ${}_2F_1(.)$ in the function $A(\delta,\alpha)$ is the hyper geometric function, while the Beta function in $C(\delta,\alpha)$ is formulated as $B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt$. We assume that there are on average K video requests from each MU within unit period, and that the average back-haul Cost for a video transmission is S^{bh}. Based on $\Pr(\varepsilon_{vf})$ in Eq. (4), we obtain the cost reduction /UAP for the back-haul Channels of L as

$$S^{BH} = \sum_{j=1}^{F}\sum_{j=2}^{V} p_{j1}\, q_{j2}\zeta\, \mathrm{K}\, \Pr(\varepsilon_{j2,j1})\, s^{bh} \quad (5)$$

By combining the above two items, the overall profit/U AP for L can be expressed as

$$S^{NSP} = S^{RT} + S^{BH} \ . \quad (6)$$

B. Average Profit of the VRs

Note that the MUs can download the videos either from the memories of the SBSs directly or from the servers of the VRs at backbone networks via back-haul channels. In the first case, the MUs will be levied by the VRs an extra amount of money in addition to the videos' prices because of the higher-rate local streaming, namely, local downloading surcharge (LDS). We assume that the LDS of each video is set as s^{ld}. Then the revenue / $U\ AP$ for a VR v_u gained from the LDS can be calculated as

$$S_v^{LD} \sum_{j=1}^{F} p_j q_v \zeta K Pr(\varepsilon_{vj}) s^{ld} . \quad (2)$$

Additionally, v_v pays for renting the SBS from L. The related cost / U AP can be written as

$$S_v^{RT} = \tau_v \lambda s_v . \quad (3)$$

Combining the above two items, the overall profit / *U AP* for L can be expressed as

$$S_v^{VR} = S_v^{LD} - S_v^{RT} \quad (4)$$

IV. PROBLEM FORMULATION

In this section, we first present the Stackelberg game formulation for our price-based SBS allocation scheme. Then the equilibrium of the proposed game is investigated.

A. Stackelberg Game Formulation

Stackelberg game is a strategic game that consists of a leader and several followers competing with each other for certain resources [20]. The leader moves first and the followers move subsequently. In our small-cell caching system, we model the NSP L *as the leader* and the *V* VRs as the followers.

The NSP imposes a price vector S $=[s_1, s_2, \dots, s_v]$ for the lease of its SBSs, where $s_v, \forall_v$ has been defined in the previous section as the price per unit period charged on v_v for renting an SBS. After the price vector s is set, the VRs updated the fraction $\tau_v, \forall_v$ that they tend to rent from L

1)Optimization Formulation of the Leader: Observe from the above game model that the NSP's objective is to maximize its profit S^{NSP} formulated in Eq (6). Note that for $\forall_v$, the fraction τ_v *is* a function of the price s_v under the Stackelberg game formulation. This means that the fraction of the SBSs that each VR is willing to rent depends on the specific price charged to them for renting an SBS. Consequently, the NSP has to find the optimal price vector s for maximizing its profit. This optimizing problem can be summarized as follows Problem1: The optimization problem of maximizing $L's$ profit can be formulated as

$$\max_{s\geq 0} S^{NSP}(s,\tau),$$

$$\text{s.t} \sum_{j=1}^{V} \tau_j \leq 1 \quad (10)$$

2) Optimization formulation of the followers: The profit gained by the VR v_u in Eq.(9) can be further written as

$$S_v^{VR}(\tau_v, s_v) = \sum_{j=1}^{F} p_j q_v \zeta\, \mathrm{K} \Pr(\varepsilon_{v,j}) s^{id} - \tau_v \lambda s_v =$$

$$\sum_{j=1}^{F} \frac{p_j q_v \zeta K S^{ld} \tau_v}{(A(\delta,\alpha)-C(\delta,\alpha)+1)\tau_v + C(\delta,\alpha)F} - \lambda s_v \tau_v . \quad (11)$$

We can see from Eq. (11) that once the price s_v is fixed, the profit of v_v depends on v_v, i.e., the fraction of SBSs that are rented by v_v. If v_v increases the fraction τ_v, it will gain more revenue by levying surcharges from more MUs, while at the same time, v_v will have to pay for renting more SBSs. Therefore,τ_v has to be optimized for maximizing the profit of v_v. This optimization can be formulated as follows.

Problem 2: The optimization problem of maximizing v_v's profit can be written as

$$\max_{\tau_v \geq 0} S_v^{VR}(\tau_v, s_v) \quad \textbf{(5)}$$

Problem 1 and *Problem 2* together form a Stackelberg game. The objective of this game is to find the Stackelberg Equilibrium (SE) points from which neither the leader (NSP) nor the followers (VRs) have incentives to deviate. In the following, we investigate the SE points for the proposed game

B. Stackelberg Equilibrium

For our stackelberg game, the SE is defined as follows.
Definition 1: let $s^* \triangleq [s_1^*, s_2^*, \dots, s_V^*]$ be a solution for problem 1, and τ_v^* be a solution for problem2, $\forall_v$. Define $\tau^* \triangleq [\tau_1^*, \tau_2^*, \dots, \tau_V^*]$. Then the point (s^*, τ^*) is an SE for the proposed stackelberg game if for any (s, τ) with $s \geq 0$ and $\tau \geq 0$, the following condition is satisfied

$$S^{NSP}(s^*, \tau^*) \geq S^{NSP}(s, \tau^*), \quad S_v^{VR}(s_v^*, \tau_v^*) \geq S_v^{VR}(s_v^*, \tau_v), \forall_v \quad \textbf{(13)}$$

Generally, the SE of a Stackelberg game can be obtained by finding its perfect Nash Equilibrium (NE). In our proposed game, we can see that the VRs strictly compete in a non-cooperative fashion. Therefore, a non-cooperative sub game on controlling the fractions of rented SBSs is formulated at the VRs' side. For a non-cooperative game, the NE is defined as the operating points at which no players can improve utility by changing its strategy unilaterally. At the NSP's side, since there is only one player, the best response of the NSP is to solve *Problem 1*. To achieve this, we need to first find the best response functions of the followers, based on which, we solve the best response function for the leader. Therefore, in our game, we first solve *Problem 2* given a price vector s. Then with the obtained best response function τ^* of the VRs, we solve *Problem 1* for the optimal price s^*. In the following, we will have an in-depth investigation on this game theoretic optimization

V. GAME THEORETIC OPTIMIZATION

In this section, we will solve the optimization problem in our game. Under the non-uniform pricing scheme, where the NSP L charges the VRs with different prices $s_1, \dots, s_V$ for renting an SBS. In this scheme, we first solve *Problem 2* at the VRs, and rewrite Eq. (11) as

$$S_v^{VR}(\tau_v, s_v) = \frac{\Gamma_v s^{ld} \tau_v}{\Theta \tau_v + \Lambda} - \lambda s_v \tau_v, 0 \quad \textbf{(6)}$$

Where $\Gamma \triangleq \sum_{j=1}^{F} p_j q_v \zeta K$, $\Theta \triangleq A(\delta, \alpha) - C(\delta, \alpha) + 1$ and $\Lambda \triangleq C(\delta, \alpha) F$. We observe that Eq.(14) is a concave function over the variable τ_v. Thus , we can obtain the optimal solution by solving the Karush-Kuhn-Tucher(KKT) conditions, and we have the following lemma. Lemma1: For given price s_v, the optimal solution of problem 2 is

$$\tau_v^* = \left(\sqrt{\frac{\Gamma_v \Lambda s^{ld}}{\Theta^2 \lambda}} \sqrt{\frac{1}{s_v}} - \frac{\Lambda}{\Theta} \right)^+, \quad \textbf{(7)}$$

Where $(.)^+ \triangleq \max(\,;\,0)$. Proof: The optimal solution τ_v^* of v_v can be obtained by deriving S_v^{VR} with respect to τ_v and solving $dS_v^{VR}=0$ under the constraint that $\tau_v \geq 0$. We can see from *Lemma 1* that, if the price s_v is set too high, i.e., $s_v \geq \frac{\Gamma_v s^{ld}}{\Lambda\lambda}$, the VR *Vv* will opt out for renting any SBS from L due the high price charged. Consequently, the VR v_v will not participate in the game. In the following derivations, we assume that the LDS on each video s^{ld} is set by the VRs to be the cost of a video transmission via back-haul channels s^{bh}. The rational behind this assumption is as follows. Since a local downloading reduce a back-haul transmission, this saved back-haul transmission can be potentially utilized to provide extra services (equivalent to the value of s^{bh}) for the MUs. In addition, the MUs enjoy the benefit from faster local video transmissions. In light of this, it is reasonable to assume that the MUs are willing to accept the price s^{bh} for a local video transmission. Substituting the optimal τ_v^* of Eq. (15) into Eq. (6) and carry out some further manipulations, we arrive at

$$S^{NSP} = \sum_{j=1}^{V} \lambda s_j \left(\sqrt{\frac{\Gamma_j \Lambda s^{bh}}{\Theta^2 \lambda}} \sqrt{\frac{1}{s_j}} - \frac{\Lambda}{\Theta} \right)^+ \quad +$$

$$\frac{\sum_{i=1}^{F} p_i q_j \zeta K s^{bh} \left(\sqrt{\frac{\Gamma_j \Lambda s^{bh}}{\Theta^2 \lambda}} \sqrt{\frac{1}{s_j}} - \frac{\Lambda}{\Theta} \right)^+}{\Theta \left(\sqrt{\frac{\Gamma_j \Lambda s^{bh}}{\Theta^2 \lambda}} \sqrt{\frac{1}{s_j}} - \frac{\Lambda}{\Theta} \right)^+ + \Lambda} \quad =$$

$$\sum_{j=1}^{V} \frac{\xi_1}{\Theta} \left(-\Lambda\lambda s_j + \left(\sqrt{s^{bh}} - \frac{s^{bh}}{\sqrt{s^{bh}}} \right) \sqrt{\Gamma_j \Lambda\lambda s_j} + \Gamma_j s^{bh} \right) \quad =$$

$$\sum_{j=1}^{V} \frac{\xi_1}{\Theta} \left(-\Lambda\lambda s_j + \Gamma_j s^{bh} \right), \quad \textbf{(8)}$$

Where ξ_j is the indicator function, with $\xi_j = 1$ if $s_j < \frac{\Gamma_j s^{bh}}{\Lambda\lambda}$ and $\xi_j = 0$ otherwise. Upon defining the binary vector $\xi \triangleq [\xi_1, \xi_2, \dots \xi_V]$, we can rewrite problem 1 as follows.Problem 3: Given the optimal solution τ_v^*, $\forall_v$, gleaned from the followers, we can rewrite problem 1 as

$$\min_{\xi,\ s \geq 0} \sum_{j=1}^{V} \xi_j (\Lambda\lambda s_j - \Gamma_j s^{bh})$$

$$s.t.\ \sum_{j=1}^{V} \xi_j \left(\sqrt{\Gamma_j \Lambda s^{bh}} - \Lambda \right) \leq \Theta \quad \textbf{(17)}$$

Observe from Eq. (17) that *Problem 3 is* non-convex due to ξ. However, for a given ξ, this problem can be solved by satisfying the KKT conditions.

VI. DISCUSSIONS OF OTHER SCHEMES

Let us now consider two other schemes, namely, an uniform pricing scheme and a global optimization scheme.

A. Uniform Pricing Scheme

In contrast to the non-uniform pricing scheme, the uniform pricing scheme deliberately imposes the same price on the VRs in the game. We denote the fixed Price by *s*. In this case, similar to *Lemma 1*, *Problem 2* can be solved by

$$\tau_v^* = \left(\sqrt{\frac{\Gamma_v \Lambda s^{bh}}{\Theta^2 \lambda}} \sqrt{\frac{1}{s}} - \frac{\Lambda}{\Theta} \right)^+. \quad \textbf{(9)}$$

Problem 4 can be converted to that of minimizing *subject to the constraint* $\sum_{j=1}^{v}\sqrt{\frac{\Gamma_j}{s}} \leq (V\Lambda+\Theta)\sqrt{\frac{\lambda}{\Lambda s^{bh}}}$.. We then obtain the optimal $\hat{s}$ for this special case as

$$\hat{s} = \frac{\Lambda s^{bh}(\sum_{j=1}^{V}\sqrt{\Gamma_j})^2}{\lambda(V\Lambda+\Theta)^2}. \quad \textbf{(10)}$$

To guarantee that all the VRs are Capable of participating in the game, ie., $\xi_v = 1, \forall v,$ with the optimal price $\frac{s^{bh}}{\Lambda\lambda}$. Then we have the following constraint on the storage Q as

$$Q > Q'\min \triangleq \frac{NC(\delta,\alpha)\left(\sum_{j=1}^{v}\sqrt{\frac{q_j}{q_v}}-V\right)}{A(\delta,\alpha)-C(\delta,\alpha)+1}. \quad \textbf{(11)}$$

We can see that we require a larger storage size *Q* in Eq. (20) to accommodate all the VRs under the non-uniform pricing scheme, since we have $\sum_{j=1}^{v}\sqrt{\frac{q_j}{q_v}} > \sum_{j=1}^{v}{}^{2}\sqrt{\frac{q_j}{q_v}}$. the optimal $s^* = [s_1^*, \ldots, s^*]$ in the uniform pricing scheme can be readily obtained as

$$;s_v^* = \begin{cases}\frac{\Lambda s^{bh}(\sum_{j=1}^{\hat{u}}\sqrt{\Gamma_j})^2}{\lambda(\hat{u}\Lambda+\Theta)^2}, & v = 1,\ldots,\hat{u},\\ \infty, & v = \hat{u}+1, \ldots, V,\end{cases} \quad \textbf{(12)}$$

Where $\hat{u} = \arg\min_u\{s_u : u = 1,2, \ldots, T\}$, **(13)**

With $s_u = \frac{u\Lambda^2 s^{bh}(\sum_{j=1}^{u}\sqrt{\Gamma_j})^2}{(u\Lambda+\Theta)^2} - \sum_{j=1}^{u}\Gamma_j s^{bh}$,

$$T = \begin{cases}1, & \overline{U}_1 < Q \leq \overline{U}_2,\\ \ldots, & \\ u, & \overline{U}_u < Q \leq \overline{U}_{u+1},\\ \ldots, & \\ V, & \overline{U}_V < Q.\end{cases} \quad \textbf{(14)}$$

Note that $\overline{U}_v$ in Eq. (23) is defined as

$$\overline{U}_v \triangleq \frac{NC(\delta,\alpha)\left(\sum_{j=1}^{v}\sqrt{\frac{q_j}{q_v}}-V\right)}{A(\delta,\alpha)-C(\delta,\alpha)+1} \quad \textbf{(24)}$$

It is clear that the uniform pricing scheme is inferior to the non-uniform pricing scheme in terms of maximizing S^{NSP}. However; we will show in the following problem that the uniform pricing scheme offers the optimal solution to maximizing the back-haul cost reduction S^{BH} at the NSP in conjunction with $\tau_v^*, \forall_v$, from the followers. *Problem 5:* With the aid of the optimal solutions $\tau_v^*, \forall v$, from the followers, the maximization on S^{BH} is achieved by solving the following problem:

$$\min_{\xi,s\geq 0}\sum_{j=1}^{v}\xi_j\left(\sqrt{s^{bh}}\sqrt{\Gamma_j\Lambda\lambda}\sqrt{s_j} - \Gamma_j s^{bh}\right),$$
$$\text{s.t } \sum_{j=1}^{v}\xi_j\left(\sqrt{\frac{\Gamma_j\Lambda s^{bh}}{\lambda s_j}}-\Lambda\right) \leq \Theta. \quad \textbf{(25)}$$

The optimal solution to *Problem 5* can be readily shown to be s^*given in Eq. (21). This proof follows the similar procedure of the optimization method presented in the previous section. Thus it is skipped for brevity. In this sense, the uniform pricing scheme is superior to the non-uniform scheme in terms of reducing more cost on back-haul channel transmissions.

B. Global Optimization Scheme

In the global optimization scheme, we are interested in the sum profit of the NSP and VRs, which can be expressed as.

$$S^{GLB} = S^{NSP} + \sum_{j=1}^{V}S_j^{VR} = \sum_{j_1}^{V}\sum_{j_2}^{F}\frac{2p_{j_2}q_{j_1}\zeta K s^{bh}\tau_{j_1}}{(A(\delta,\alpha)-C(\delta,\alpha)+1)\tau_{j_1}+C(\delta,\alpha)F}$$
$$=2S^{BH}. \quad \textbf{(26)}$$

Observe from Eq. (26), we can see that the sum profit S^{GLB} is twice the back-haul cost reduction S^{BH}, where the vector $\boldsymbol{\tau}$ is the only variable of this maximization problem.

Problem 8:

The optimization of the sum profit *SGLB* can be formulated as

$$\min_{\tau\geq 0} \sum_{j=1}^{V}\frac{\tau_{j_1}\sum_{j_1}^{F}p_{j_2}q_{j_1}\zeta K s^{bh}}{(A(\delta,\alpha)-C(\delta,\alpha)+1)\tau_{j_1}+C(\delta,\alpha)F},$$
$$\text{s.t. } \sum_{j=1}^{v}\tau_j \leq 1. \quad \textbf{(15)}$$

Problem 6 is a typical water-filling optimization problem. By relying on the classic Lagrangian multiplier, we arrive at the optimal solution

$$\hat{\tau}_v = \left(\frac{\frac{\sqrt{q_v}}{\eta}-C(\delta,\alpha)F}{A(\delta,\alpha)-C(\delta,\alpha)+1}\right)^+, \forall v, \quad \textbf{(16)}$$

Where we have $\eta = \frac{\sum_{j=1}^{\bar{v}}\sqrt{q_j}}{\bar{v}\,C(\delta,\alpha)F+A(\delta,\alpha)-C(\delta,\alpha)+1}$ and $\bar{v}$ satisfies the constraint of $\bar{\tau}_{v>0}$.

C. Comparisons

Let us now compare the optimal SBS allocation variable $\tau_v$in the context of the above two schemes. First, we investigate τ_v^* in the uniform pricing scheme. By substituting Eq. (21) into Eq. (18), we have

$$\tau_v^* = \left(\frac{\sqrt{\Gamma_v\Lambda s^{bh}}}{\Theta^2\lambda}\sqrt{\frac{1}{s_v^*}}-\frac{\Lambda}{\Theta}\right)^+$$
$$= \begin{cases}\frac{\frac{\sqrt{q_v}}{\eta}-C(\delta,\alpha)F}{A(\delta,\alpha)-C(\delta,\alpha)+1}, & v = 1,\ldots,\hat{u}\\ 0 & v = \hat{u}+1,\ldots,V,\end{cases} \quad \textbf{(17)}$$

Where $\eta' = \frac{\sum_{j=1}^{\bar{v}}\sqrt{q_j}}{\bar{v}\,C(\delta,\alpha)F+A(\delta,\alpha)-C(\delta,\alpha)+1}$ and $\hat{u}$ ensures $\tau_v^* > 0$. Then, comparing τ_v^*given in Eq. (29) to the optimal solution $\hat{\tau}$ of the global optimization scheme given by Eq. (28), we can see that these two solutions are the same. In other words, the uniform pricing scheme in fact represents the global optimization scheme in terms of maximizing the sum profit $S^{GLB}$and maximizing the back-haul cost reductionS^{BH}.

VII. SIMULATION RESULTS AND OBSERVATIONS

In this section, we provide both numerical as well as Monte-Carlo simulation results for evaluating the performance of the proposed schemes. The physical layer parameters of our Simulations, such as the path-loss exponent

α, transmit power P of the SBSs and the noise power σ^2 are similar to those of the 3GPP standards. The unit of noise power and transmit power is Watt, while the SBS and MU intensities are expressed in terms of the numbers of the nodes per square kilometer. We set the path-loss exponent to α = 4, the SBS transmit power to P = 1Watt, the noise power to σ^2 =$10^{-10}$$watt$, and the pre-set SINR threshold to δ=0.01. For the file caching system, we set the number of files in F to N = 500 and set the number of VRs to V = 15. For the network deployments, we set the intensity of the MUs to $\zeta = 50/km^2$, and investigate three cases of the SBS deployments as $\lambda = 5/km^2, 10/km^2, 40/km^2$. For the pricing system, the profit=UAP is considered to be the profit gained per month within an area of one square kilometer, i.e., */month.km*^{2}. We note that the profits gained by the NSP and by the VRs are proportional to the cost s^{bth} of back-haul channels for transmitting a video. Hence, without loss of generality, we set s^{bth} for simplicity. Additionally, we set K = 10/*month*, which is the average number of video requests from an MU per month. We first verify our derivation of Pr (ε_v, f) by comparing the analytical results of *Theorem 1* to the Monte-Carlo simulation results. Upon verifying $\Pr(\varepsilon_v, f)$, we will investigate the optimization results within the framework of the proposed Stackelberg game by providing numerical results. Comparisons between the simulations and analytical results on Pr (ε_v, f). We consider four kinds of storage size Q in each SBS, i.e.,

$Q = 10, 50, 100, 500$ and three kinds of SBS intensity,

ie $\lambda = 5, 20, 40$

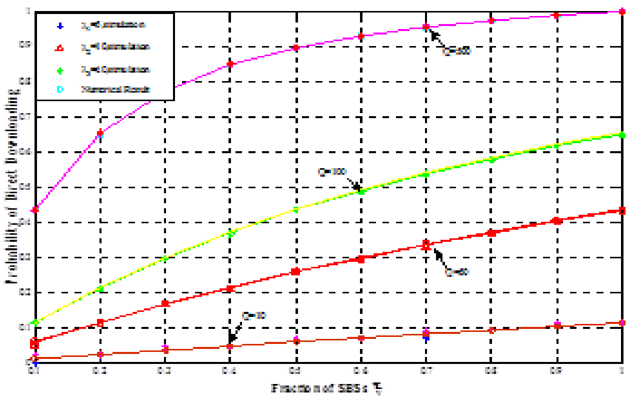

Fig. 1. Comparing simulations and analytical results on

Pr (ε_v , f) .We consider four kinds of storage size Q in each SBS, i.e.,Q = 10, 50, 100, 500 and three kinds of SBS intensity, ie., λ = 5, 20, 40

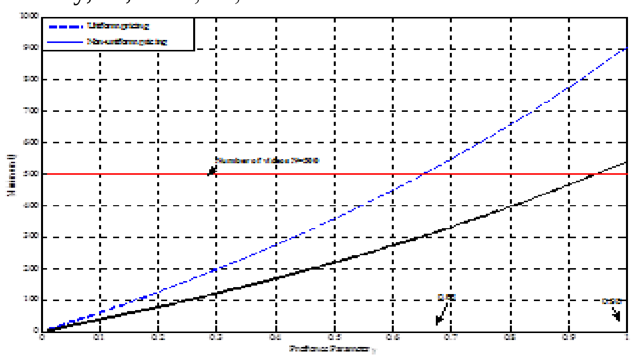

Fig2. The minimum number of Q that allows all the VRs to participate in the game under different parameter γ.

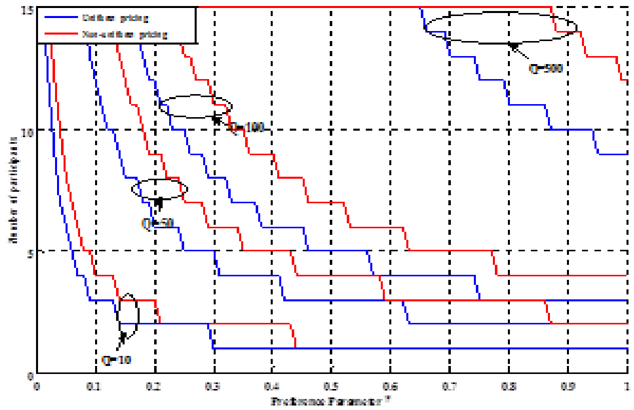

Fig. 3. Number of participants, i.e., the VRs that are in the game, vs. the preference parameter γ.

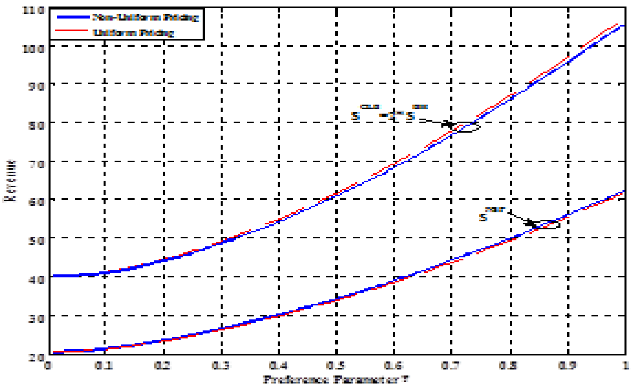

Fig.4. various revenues, including S^{NSP} and S^{BH} vs. the preference parameter γ, under the two scheme.

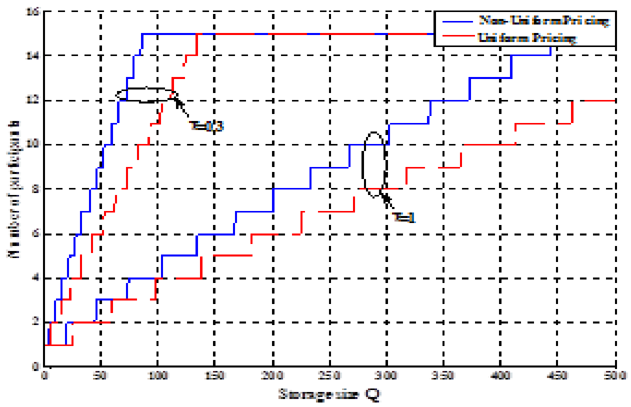

Fig. 5. Number of participants vs. the storage size Q

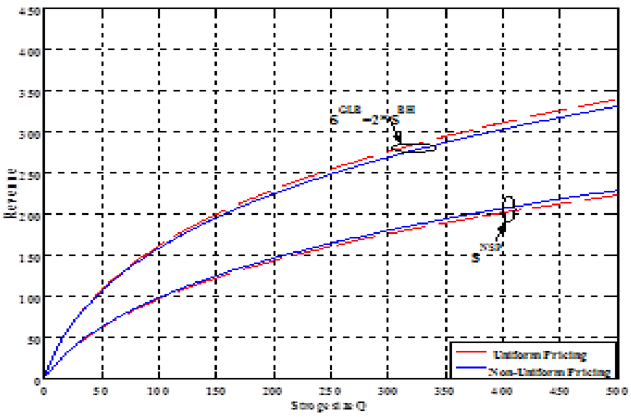

Fig.6.Various revenues, including S^{NSP} and S^{GLB} vs. the storage size Q under the two schemes.

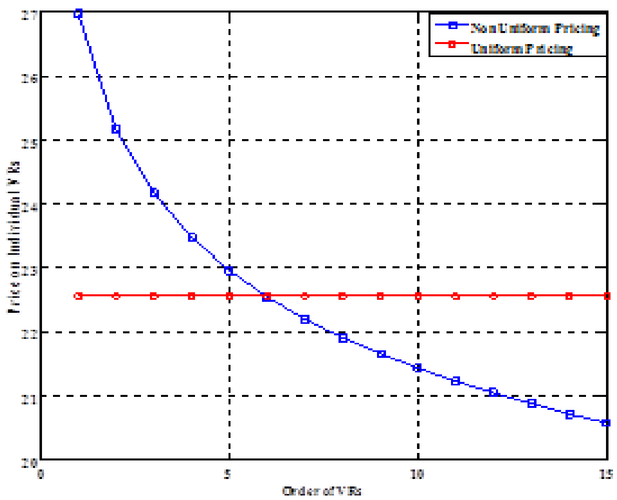

Fig. 7. Price charged on each VR for renting an SBS per month.

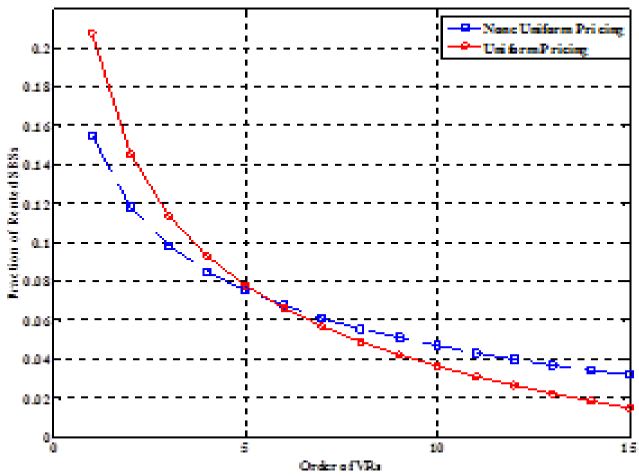

Fig. 8. The fraction of SBSs that are rented by each VR.

In Fig.1 .we can see that simulation results closely match the analytical results in Theorem 1 . Our simulation shows that the intensity λ does not affect Pr (ε_{v}, f), which is consistent with our analytic results. Furthermore, a larger Q leads to a higher value of Pr (ε_{v}, f), . Hence enlarging the storage size is helpful for achieving a higher probability of direct downloading.

Fig. 2 .We can also observe that for $\gamma > 0.66$ in the UPS and for $\gamma > 0.98$ in the NUPS, the Minimum Q becomes larger than the overall number of videos N. In both cases, since we have $Q \leq N(Q$ results in the same performance as $Q = N)$ some unpopular VRs will be excluded from the game.

Fig.3 shows that the number of VR participants keeps going down upon increasing γ in the both schemes. The NUPS always keeps more VRs in the game than the UPS under the same. At the same time, by considering $Q = 10, 50, 100, 500$,it is shown that for a given γ, a higher Q will keep more VRs in the game.

Fig.4. shows two kinds of revenues gained by the two Schemes for a given storage of $Q = 500$ namely, the global profit S^{GLB} defined in Eq. (26) and the profit of the NSP defined in Eq. (6). Recall that we have $S^{GLB} = 2S^{BH}$ according to Eq. (26). We can see that the revenues of both schemes increase exponentially upon increasing, as stated in *Remark 4*. As our analytical result shows, the profit S^{NSP} gained by the NUPS is optimal and thus it is higher than that gained by the UPS, while the UPS maximizes both S^{GLB} and S^{BH} fig.4 verifies the accuracy of our derivations.

Fig.5. shows the number of participants in the game versus Q, where $\gamma = 0.3$ and 1 are considered. It is shown that for a larger Q more VRs are able to participate in the game. Again, the NUPS outperforms the UPS owing to its capability of accommodating more VRs for a given Q . By comparing the scenarios of $\gamma = 0.3$ *and* 1, we find that for $\gamma = 0.3$ a given increase of Q can accommodate more VRs in the game than $\gamma = 1$.

Fig. 6. We can see that the revenues of both schemes increase with the growth of Q It is shown that the profit S^{NSP} gained by the NUPS is higher than the one gained by the UPS, while the UPS outperforms the NUPS in terms of both S^{GLB} and S^{BH}.

Fig.7. We can see that in the NUPS, the price for renting an SBS is higher for the VRs having a higher popularity than those with a lower popularity. By contrast, in the UPS, this price is fixed for all the VRs .

In Fig.8. We observe that in both schemes, the VRs associated with a high popularity tend to rent more SBSs. Furthermore, the UPS seems more aggressive than the NUPS, since the less popular VRs of the UPS are more difficult to rent an SBS, and thus these VRs are likely to be excluded from the game with a higher probability.

VIII. CONCLUSIONS

In this paper, we considered a marketed small-cell caching system comprising of an NSP and multiple VRS. In such a system, the NSP leases its SBSs to the VRs to gain profits, and for reducing the costs of back-haul channel transmissions, while the VRs, after storing popular videos to the rented SBSs, can provide faster transmissions to the MUs,. We proposed a Stackelberg game theoretic framework by viewing the SBSs as a type of resources. We first modeled the MUs and SBSs using two independent PPPs via stochastic geometry, and developed the probability expression of direct downloading. Then, based on the probability, we formulated the Stackelberg game to maximize the average profit of the NSP as well as individual VRs. Next, we investigated the Stackelberg equilibrium by solving the associated non-convex optimization problem. We proved that the non-uniform pricing scheme can effectively maximize the profit of the NSP, while the uniform one maximizes the sum profit of the NSP and we also verified monte-carlo simulations that the direct downloading probability under our PPP model is consistent with our derived results. Simulation results were provided to show that the proposed scheme is effective in pricing and resource allocation.

REFERENCES

[1] N. Golrezaei, A. Molisch, A. Dimakis, and G. Caire, "Femtocaching and device-to-device collaboration: A new architecture for wireless video distribution," *IEEE Commun. Mag.*, vol. 51, no. 4, pp. 142–149, Apr.2013

[2] X. Wang, M. Chen, T. Taleb, A. Ksentini, and V. Leung, "Cache in the air: Exploiting content caching and delivery techniques for 5G systems,"*IEEE Commun. Mag.*, vol. 52, no. 2, pp. 131–139, Feb. 2014.

[3] M. Maddah-Ali and U.Niesen, "Decentralized coded caching attains order-optimal memory-rate tradeoff," in *51st Annual Allerton Conference on Communication, Control, and Computing (Allerton)*, Oct. 2013, pp.421–427.

[4] N. Golrezaei, P. Mansourifard, A. Molisch, and A. Dimakis, "Basestation assisted device-to-device communications for high-throughput wireless video networks," *IEEE Trans. Wireless Commun.*, vol. 13, no. 7,pp. 3665–3676, July 2014.

[5] M. Ji, G. Caire, and A. F. Molisch, "Wireless device-to-device caching networks: Basic principles and system performance," *arXiv preprint arXiv:1305.5216*, May 2013.

[6] M. Ji, G. Caire, and A. Molisch, "Optimal throughput-outage trade-off in wireless one-hop caching networks," in *IEEE International Symposium on Information Theory Proceedings (ISIT)*, Jul. 2013, pp. 1461–1465.

[7] P. Gupta and P. Kumar, "The capacity of wireless networks," *IEEE Trans.Inform. Theory*, vol. 46, no. 2, pp. 388–404, Mar. 2000.

[8] F. Boccardi, R. Heath, A. Lozano, T. Marzetta, and P. Popovski, "Five disruptive technology directions for 5G," *IEEE Commun. Mag.*, vol. 52,no. 2, pp. 74–80, Feb. 2014.

[9] A. Damnjanovic, J. Montojo, Y. Wei, T. Ji, T. Luo, M. Vajapeyam,T. Yoo, O. Song, and D. Malladi, "A survey on 3GPP heterogeneous networks," *IEEE Wireless Commun.*, vol. 18, no. 3, pp. 10–21, Jun. 2011.

[10] J. Akhtman and L. Hanzo, "Heterogeneous networking: An enabling paradigm for ubiquitous wireless communications," *Proceedings of the IEEE*, vol. 98, no. 2, pp. 135–138, Feb. 2010.

[11] S. Bayat, R. Louie, Z. Han, B. Vucetic, and Y. Li, "Distributed user association and femtocell allocation in heterogeneous wireless networks," *IEEE Trans. Commun.*, vol. 62, no. 8, pp. 3027–3043, Aug. 2014.

[12] M. Mirahmadi, A. Al-Dweik, and A. Shami, "Interference modeling and performance evaluation of heterogeneous cellular networks," *IEEE Trans. Commun.*, vol. 62, no. 6, pp. 2132–2144, Jun. 2014.

[13] A. Gupta, H. Dhillon, S. Vishwanath, and J. Andrews,"Downlink multiantenna heterogeneous cellular network with load balancing," *IEEE Trans. Commun.*, vol. 62, no. 11, pp. 4052–4067, Nov. 2014.

[14] M. Liebsch, S. Schmid, and J. Awano, "Reducing backhaul costs for mobile content delivery-An analytical study," in *IEEE International Conference on Communications (ICC)*, Jun. 2012, pp. 2895–2900.

[15] K. Shanmugam, N. Golrezaei, A. Dimakis, A. Molisch, and G. Caire, "Femtocaching: Wireless content delivery through distributed caching helpers," *IEEE Trans. Inform. Theory*, vol. 59, no. 12, pp. 8402–8413,Dec. 2013.

[16] E. Bastug, M. Bennis, and M. Debbah, "Cache-enabled small cell networks: Modeling and tradeoffs," in *11th International Symposium on Wireless Communications Systems (ISWCS)*, Aug. 2014, pp. 649–653.

[17] D. Stoyan, W. Kendall, and M. Mecke, *Stochastic Geometry and Its Applications*. Second Edition, John Wiley and Sons, 2003.

[18] M. Haenggi, J. Andrews, F. Baccelli, O. Dousse, and M. Franceschetti, "Stochastic geometry and random graphs for the analysis and design of wireless networks," *IEEE J. Select. Areas Commun.*, vol. 27, no. 7, pp.

[19] G. Vazquez-Vilar, C. Mosquera, and S. Jayaweera, "Primary user enters the game: Performance of dynamic spectrum leasing in cognitive radio networks," *IEEE Trans. Wireless Commun.*, vol. 9, no. 12, pp. 3625–3629, Dec. 2010.

[20] D. Fudenberg and J. Tirole, *Game Theory*. MIT Press, 1993